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## B.M.S. COLLEGE FOR WOMEN, AUTONOMOUS

BENGALURU – 560004

SEMESTER END EXAMINATION – SEPT/OCT 2023

M.Sc in Mathematics – 4<sup>th</sup> Semester

### SPECIAL FUNCTIONS

Course Code: MM404T

Duration: 3 Hours

QP Code: 14004

Max marks: 70

*Instructions:* 1) All questions carry equal marks.  
2) Answer any five full questions.

1. (a) Prove that the hypergeometric function  ${}_2F_1(a, b; c; z)$  satisfies the Gauss equation in the region  $|z| < 1$ .  
(b) Derive the integral representation for hypergeometric series  ${}_2F_1(a, b; c; z)$  and hence deduce Gauss summation formula. (7+7)
2. (a) State and prove Pfaff-Kummer summation formula.  
(b) State and prove  $q$ -binomial theorem. (7+7)
3. (a) State and prove Heine's transformation formula.  
(b) Prove that  ${}_2\phi_1(a, b; c; q, z) = \frac{(az; q)_\infty}{(z; q)_\infty} \sum_{n=0}^{\infty} \frac{(a, c; q)_n}{(q, c, az; q)_n} (-bz)^n q^{\binom{n}{2}}$ . (7+7)
4. State and prove Ramanujan's  ${}_1\psi_1$  summation formula and hence deduce Jacobi's triple product identity. (14)
5. Prove the following.
  - (i)  $\varphi(q) + \varphi(-q) = 2\varphi(q^4)$
  - (ii)  $\varphi(q) - \varphi(-q) = 4q\psi(q^8)$
  - (iii)  $\varphi(q)\varphi(-q) = \varphi^2(-q^2)$
  - (iv)  $\psi(q)\psi(-q) = \psi(q^2)\varphi(-q^2)$
  - (v)  $\varphi^2(q) + \varphi^2(-q) = 2\varphi^2(q^2)$
  - (vi)  $\varphi^2(q) - \varphi^2(-q) = 8q\psi^2(q^4)$(14)
6. Prove the following.
  - (i)  $f(a, b) + f(-a, -b) = 2f(a^3b, ab^3)$
  - (ii)  $f(a, b) - f(-a, -b) = 2af\left(\frac{b}{a}, \frac{a}{b}a^4b^4\right)$
  - (iii)  $f^2(a, b) + f^2(-a, -b) = 2f(a^2, b^2)\varphi(ab)$
  - (iv)  $f^2(a, b) - f^2(-a, -b) = 4af\left(\frac{b}{a}, \frac{a}{b}a^2b^2\right)\psi(a^2b^2)$(14)

7. (a) Define partition function  $p(n)$ . Derive the generating function for  $p(n)$ .  
(b) Prove the following:

$$(i) (1+x)(1+x^3)(1+x^5)\cdots = \sum_{n=0}^{\infty} \frac{x^{n^2}}{(x^2; x^2)_n}$$
$$(ii) (1+x^2)(1+x^4)(1+x^6)\cdots = \sum_{n=0}^{\infty} \frac{x^{n(n+1)}}{(x^2; x^2)_n}.$$

8. State and prove Rogers-Ramanujan identities.

(7+7)

(14)

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